

place many statisticians, both practising and those more theoretically inclined, in their debt. The visual appearance and general presentation of the material are excellent. Perhaps one very minor flaw is that since  $L(h, k, r) = L(k, h, r)$ , tables of  $L(h, k, r)$  for  $h \geq k$  would have been sufficient. However, this is a flaw (if at all) from the point of view of economics, but hardly so from the point of view of the user of the tables! The cost of the book is remarkably low.

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**56[K].**—E. NIEVERGELT, "Die Rangkorrelation  $U$ ," *Mitteilungsblatt für Math. Stat.*, v. 9, 1957, p. 196–232.

In contrast to Spearman's rank correlation  $R$  and Kendall's coefficient  $T$ , the author studies the van de Waerden coefficient  $U$ . Let  $p_i$  and  $q_i$  ( $i = 1, 2, 3 \dots n$ ) be the ranks of  $n$  observations on two variables  $x$  and  $y$ , and let  $\xi_i, \eta_i, \zeta_i$  be the inverses of the normal probabilities:  $F(\xi_i) = p_i/(n+1)$ ;  $F(\eta_i) = q_i/(n+1)$ ;  $F(\zeta_i) = i/(n+1)$ . Then,  $U$  is defined by  $U = \sum_{i=1}^n \xi_i \eta_i / \sum_{i=1}^n \zeta_i^2$ .

If  $x$  and  $y$  are independent the expectation of  $U$  is zero and its standard deviation is  $\sigma_U = (n-1)^{-1/2}$ , as for Spearman's coefficient. The author calculates also the 4th and 6th moments of  $U$  and  $R$ , which differ. For  $n$  large,  $U$  is asymptotically normally distributed about mean zero with standard deviation  $\sigma_U$ . The distribution of  $U$  (to 4D) is tabulated completely for  $n = 4$ , and over the upper 5% tail for  $n = 5, 6, 7$ . For larger values of  $n$  the Gram-Charlier development is used. Tables testing independence based on 5%, 2.5%, 1%, and .5% probabilities are given to 3D for  $n = 6(1) 30$ . For  $n > 30$  the normal probability function can be used.

In the case of dependence the correlation between  $R$  and  $U$  decreases slowly with  $n$  increasing. If  $x$  and  $y$  are normally distributed with zero mean, unit standard deviation and correlation  $\rho$  a generalization  $U^*$  of  $U$  to the continuous case leads to  $U^* = \rho$ . A consistent estimate for  $\rho$  is given. The  $U$  test is more powerful than the  $R$  test.

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**57[K].**—D. B. OWEN & D. T. MONK, *Tables of the Normal Probability Integral*, Sandia Corporation Technical Memorandum 64-57-51, 1957, 58 p., 22 x 28 cm. Available from the Office of Technical Services, Dept. of Commerce, Washington 25, D. C., (Physics (TID-4500, 13th Edn.), Price \$40.

The following forms of the normal probability integral

$$G(h) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^h e^{-t^2/2} dt, h \geq 0$$

$$G(-h) = 1 - G(h)$$

are given for  $h = 0(.001) 4(.01) 7$ , to 8D. For those having frequent use of  $G(h)$  these tables eliminate the simple yet troublesome computation necessary when

using the more comprehensive tables prepared at the New York Mathematical Tables Project [1] for

$$F(h) = \frac{1}{\sqrt{2\pi}} \int_{-h}^h e^{-t^2/2} dt$$

The present tables are more comprehensive than the Pearson and Hartley tables [2] for  $G(h)$  up to  $h = 7$ . These tables have been checked by the authors against other tables. (The reviewer could undertake no systematic checking, but such occasional checks as were made revealed no errors.)

Computations (made on a CRC-102A digital computer) were facilitated by using the following continued fractions:

$$R_1(x) = \frac{x}{1 - \frac{x^2}{3 + \frac{2x^2}{5 - \frac{3x^2}{7 + \dots}}}} \quad , \quad h \leq 2.5;$$

$$R_2(x) = \frac{1}{x + \frac{1}{x + \frac{2}{x + \frac{3}{x + \dots}}}} \quad , \quad h \geq 2.5.$$

Then  $G(h)$  was computed from

$$G(h) = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} e^{-h^2/2} R_1(h), \quad \text{or} \quad 1 - \frac{1}{\sqrt{2\pi}} e^{-h^2/2} R_2(h).$$

For large values of  $h$ ,  $G(-h)$  can be obtained from the formula

$$G(-h) = \frac{1}{\sqrt{2\pi}} e^{-h^2/2} M(h), \quad h \geq 0,$$

where

$$M(h) = e^{-h^2/2} \int_h^\infty e^{-t^2/2} dt.$$

$M(h)$ , Mill's ratio [3], provides constants  $a$ , an integer, and  $b$  ( $0.1 < b < 1$ ) such that

$$G(-h) = b \cdot 10^{-a}$$

For large  $h = 50(1) 150(5) 500$ ,  $M(h)$  and  $b$  are tabulated to 8S;  $a$ , of course, exactly.

The authors checked  $-\log_{10} G(-h)$  given in [2] for large  $h$ , and found one discrepancy, namely,  $-\log_{10} G(-500)$  should read 54289.90830.

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1. *Tables of Normal Probability Functions*, National Bureau of Standards Applied Math. Series, No. 23, U. S. Government Printing Office, Washington, D. C., 1953.